Team Round

30 minutes | 15 problems

1. On Venus, an extinct alien civilization wrote this message on a cave wall: If $a \star b = (7a - b^2 + 3a)$, what is $(13 \star (5 \star 8)) \div 2$?

2. The Moon rotates as it orbits Earth in such a way that a viewer on Earth only sees one face of the Moon. Given that the moon orbits Earth once every 30 days, determine how fast the moon rotates in degrees/day.

3. An alien language uses base b numerals. The string "121" in base b equals 81 in decimal. What is b?

4. Zoe and Tara are kidnapped by evil aliens and held in two separate prison cells. They're told that they will be released if they can figure out the product of their prison cell numbers. Zoe's cellmate gives them the hint that the sum of the squares of the two numbers is 110. Tara's cellmate gives them the hint that the sum of the reciprocals is 2. What is the number they need to figure out?

5. In an alternate universe, Cooper has realized that Dr. Mann plans to betray the team that brought him back. In return, he decides to screw over Dr. Mann in the most diabolical way possible. Cooper swaps Dr. Mann's fuel canisters with 15 replicas, some of which are explosive; and in a final act of pettiness, even ties Mann's oxygen generator to an absurd riddle.
   It reads, "Of the 15 fuel canisters I left you, an exact amount of N are safe.
   N can be defined by this: $((\frac{1}{2} - \frac{1}{1}) + (\frac{2}{3} - \frac{1}{2}) + (\frac{3}{4} - \frac{2}{3}) + ... + (\frac{20}{21} - \frac{19}{20})) + \frac{1}{21}$."
   How many of Dr. Mann's fuel canisters are safe?

6. One of the Trisolarans' sophons in the Three Body Problem is coming to destroy an expensive particle accelerator - the one thing that can save it is finding the units digit of the sum $9^{2364} + 7^{2357}$ to plug into a launchpad that will trigger an electric-field shielded dome. Hurry!

7. On Mars, gravity is only 40% of Earth's gravity. A sphere placed on Earth experiences a force of 10 N due to gravity. The sphere is then moved to Mars, and its radius is doubled (keeping density the same). What force does it experience?

8. Explorers find a perfectly regular triangular pyramid on the planet TOI-756 B (a regular tetrahedron). Each edge measures 42 meters. What is the volume of the pyramid? Express your answer in the simplest radical form.

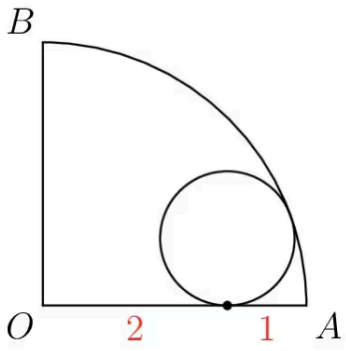
9. On Saturn, the height of a rocket can be modeled by the equation $h(t) = -3t^2 + 24t + 45$, where $h(t)$ is in meters and t is time in seconds after launch. What is the maximum altitude the rocket reaches?

10. Due to having three suns in their solar system instead of 1, Trisolarans count in base 3 instead of in base 10. What's the number 201_3 (that's 201 in base 3) in base 10, the normal system that humans use?

11. A gravitational field produces a conical distortion around a probe. The cone's height is 90 m, and its slant height is 93 m. What is the radius of the cone's base? Express your answer in the simplest radical form.

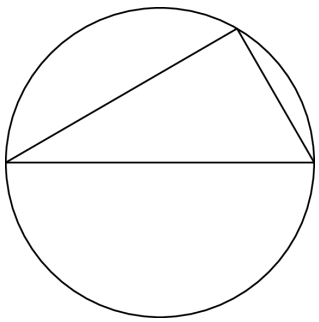
12. The speed, in kilometers per seconds, of an asteroid nearing a black hole can be modeled by the quadratic equation $x^2 + ax + 40$. If the equation's vertex is at (-4, 24) and the asteroid passes the event horizon of the black hole at x = 18, what speed will it be going when it passes the event horizon?

13. A circular orbit lies inside a larger orbit, as shown below. The smaller orbit path is tangent to side AO and arc AB. Find the radius of the smaller orbit. Express your answer as a common fraction in lowest terms.



14. Spicy Air Co. has created a monopoly on breathing substances on Pluto. They have a total of $2^{17} + 3^{17}$ tanks of spicy air, and want to split this total into groups of 5. How many items would be left ungrouped after the maximum number of items are grouped together?

15. Believe it or not, the Earth is actually a flat circle. Scientists draw a triangle on the Earth with one side as the diameter and the third vertex being on the edge of the circle. Given that the area of the triangle is $\frac{1}{4}$ the total area of the Earth, the ratio of the longer leg to the shorter leg can be expressed as $\frac{a+\sqrt{b}}{c}$, where a and c have no divisors in common and b is not divisible by the square of any prime. What is $a + b - c^2$?



(not to scale)